MENSURATION

Definition

1. **Mensuration** : It is a branch of mathematics which deals with the lengths of lines, areas of surfaces and volumes of solids.

2. **Plane Mensuration** : It deals with the sides, perimeters and areas of plane figures of different shapes.

3. **Solid Mensuration** : It deals with the areas and volumes of solid objects.

**Important Formulae**

**Right Angled Triangle** :

\[(AC)^2 = (AB)^2 + (BC)^2\]

or, \[h^2 = p^2 + b^2\]

If \(AC = 5m, AB = 4m\) then

\[(BC)^2 = (AC)^2 - (AB)^2\]

\[= 25 - 16 = 9\]

\[\therefore BC = 3m\]

**Rectangle** : A rectangle is a plane, Whose opposite sides are equal and diagonals are equal. Each angle is equal to 90°.

Here \(AB = CD;\) length \(l = 4m\)

\(AD = BC;\) breadth \(b = 3m\)

1. **Perimeter of a rectangle** = 2(length + breadth)

   \[= 2(l + b)\]

   \[= 2(4 + 3) = 14\ m\]

2. **Area of rectangle** = length \(\times\) breadth = \(l \times b = 4 \times 3\)

   \[= 12\ m^2\]
3. Length of a rectangle : \( \frac{\text{area}}{\text{breadth}} = \frac{A}{b} = \frac{12}{3} = 4 \text{ m} \)

or, \( \left( \frac{\text{perimeter}}{2} - \text{breadth} \right) = \left( \frac{14}{2} - 3 \right) = 4 \text{ m} \)

Breadth of a rectangle : \( \frac{\text{area}}{\text{length}} = \frac{A}{l} = \frac{12}{4} = 3 \text{ m} \)

or, \( \left( \frac{\text{perimeter}}{2} - \text{length} \right) = \left( \frac{14}{2} - 4 \right) = 3 \text{ m} \)

4. Diagonal of rectangle : \( \sqrt{(\text{length})^2 + (\text{breadth})^2} \)

\[ = \sqrt{l^2 + b^2} = \sqrt{4^2 + 3^2} \]

\[ = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ m} \]

**Square** : A square is a plane figure

Bounded by four equal sides having all its angles as right angles.

Here \( AB = BC = CD = AD = 5 \text{ m} = a(\text{Let}) \)

1. Perimeter of square = \( 4 \times \text{sides} = 4a \)

\[ = 4 \times 5 = 20 \text{ m} \]

2. Area of a square = \( (\text{sides})^2 = a^2 = (5)^2 = 25 \text{ sq. m} \)

3. Side of a square = \( \sqrt{\text{area}} = \sqrt{25} = 5 \text{ m} \) or,

\[ \frac{\text{Perimeter}}{4} = \frac{20}{4} = 5 \text{ m} \]

4. Diagonal of a square = \( \sqrt{2} \times \text{side} = \sqrt{2} \times a \)

\[ = \sqrt{2} \times 5 = 5\sqrt{2} \text{ m} \]

5. Side of a square = \( \frac{\text{diagonal}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5 \text{m} \)

**Triangle** :

1. Area of triangle = \( \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times h \)

\[ = \frac{1}{2} \times 15 \times 12 = 90 \text{ sq. cm} \]

here \( AD = 12 \text{ cm} = \text{height}, \ BC = 15 \text{ cm} = \text{base} \)
2. Semi perimeter of a triangle

\[ S = \frac{a+b+c}{2} = \frac{10+8+6}{2} = 12 \text{ cm} \]

here BC = a, AC = b, AB = c

3. Area of triangle = \( \sqrt{s(s-a)(s-b)(s-c)} \)

where \( a = 10 \text{cm}, b = 8 \text{cm}, c = 6 \text{cm}, s = 12 \text{cm} \)

\[ = \sqrt{12(12-10)(12-8)(12-6)} \]

\[ = \sqrt{12 \times 2 \times 4 \times 6} = 24 \text{ cm}^2 \]

4. Perimeter of a triangle = \( 2s = (a + b + c) \)

\[ = 10 + 8 + 6 = 24 \text{ cm} \]

5. Area of an equilateral triangle = \( \frac{\sqrt{3}}{4} \times (\text{side})^2 \)

\[ = \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2 \]

\[ = \frac{\sqrt{3}}{4} \times 48 = 12\sqrt{3} \text{ cm}^2 \]

6. Height of an equilateral triangle = \( \frac{\sqrt{3}}{2} \times (\text{side}) = \frac{\sqrt{3}}{2} \times 4\sqrt{3} \)

\[ = 6 \text{ cm} \]

7. Perimeter of an equilateral triangle = \( 3 \times (\text{side}) \)

\[ = 3 \times 4\sqrt{3} = 12\sqrt{3} \text{ cm} \]

**Quadrilateral :**

**Parallelogram :**

(i) Area of parallelogram = base \times height

\[ = b \times h \]

\[ = 8 \times 5 = 40 \text{ sq.cm.} \]

(ii) Perimeter of a parallelogram = \( 2(AB + BC) \)
\[
2(8 + 5) = 26 \text{ cm}
\]

**Rhombus:**

(i) **Area of rhombus** = \(\frac{1}{2} \times \text{product of diagonals}\)

\[
= \frac{1}{2} (d_1 \cdot d_2) = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2
\]

(ii) **Perimeter of rhombus** = \(4 \times \text{side} = 4a\)

here \(AB = BC = CD = AD = 4a\)

\(AC = d_1, \ BD = d_2\)

**Trapezium:**

(i) **Area of a trapezium** = \(\frac{1}{2} \times \text{(sum of parallel sides)} \times \text{height}\)

\[
= \frac{1}{2} \times (a + b) \times h
\]

\[
= \frac{1}{2} \times (15 + 17) \times 10
\]

\[
= \frac{1}{2} \times 32 \times 10 = 160 \text{ cm}^2
\]

**Regular Hexagon:**

(i) **Area of a regular hexagon** = \(6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2\)

(ii) **Perimeter of a regular hexagon** = \(6 \times \text{side}\)

**Circle:**

(i) **Circumference of a circle** = \(\pi \times \text{diameter}\)

\[
= \pi \times 2r = 2\pi r
\]
\[ 2 \times \frac{22}{7} \times 42 = 264 \text{ cm} \]

(ii) Radius of a circle = \( \frac{\text{circumference}}{2\pi} = \frac{264 \times 7}{2 \times 22} = 42 \text{ cm} \)

(iii) Area of a circle = \( \pi \times r^2 = \frac{22}{7} \times 42^2 = \frac{22}{7} \times 42 \times 42 = 5544 \text{ cm}^2 \)

(iv) Radius of a circle = \( \sqrt{\frac{\text{area}}{\pi}} \)

\[ = \sqrt{\frac{5544}{22}} \times 7 = \sqrt{1764} = 42 \text{ cm} \]

(v) Area of a semi circle = \( \frac{1}{2} \pi r^2 = \frac{1}{8} \pi d^2 \)

\[ = \frac{1}{2} \times \frac{22}{7} \times 42^2 = 2772 \text{ cm}^2 \]

(vi) Circumference of semi circle = \( \frac{22}{7} \times 42 = 132 \text{ cm} \)

(vii) Perimeter of semi circle = \( (\pi r + 2r) = (\pi + 2) r = (\pi + 2) \frac{d}{2} \)

(viii) Area of sector OAB = \( \frac{x}{360} \times \pi r^2 \)

(x being the central angle)

\[ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 3.5 \times 3.5 = 3.21 \text{ sq. m.} \]

(ix) Central angle by arc AB = \( 360^\circ \times \frac{\text{area of OAB}}{\text{area of circle}} \)

\[ = 360^\circ \times \frac{3.21}{\frac{22}{7} \times 3.5 \times 3.5} = \frac{360 \times 321}{22 \times 35 \times 5} = 30^\circ \text{(approx)} \]

(x) Radius of circle = \( \sqrt{\frac{360^\circ}{\text{central angle by arc}} \times \frac{\text{area of OAB}}{\pi}} \)

\[ = \sqrt{\frac{360^\circ}{30^\circ} \times \frac{3.21}{\frac{22}{7}}} = \sqrt{\frac{134.82}{11}} = \sqrt{12.23} = 3.5 \text{ m.} \]

(xi) Area of ring

= difference of the area of two circle
= πR² − πr² = (R² − r²)
= π(R + r)(R − r)
= (sum of radius)(diff. of radius)
= \frac{22}{7} \times (4 + 3)(4 − 3) = \frac{22}{7} \times 7 \times 1
= 22 \text{ sq. cm.}

**Cuboid and Cube :**

(i) Total surface area of cuboid
= 2(lb + bh + hl) sq. unit
Here l = length, b = breadth, h = height
= 2(12 \times 8 + 8 \times 6 + 6 \times 12) = 2(96 + 48 + 72) = 2 \times 216 = 432 \text{ sq. cm.}

(ii) Volume of a cuboid = (length × breadth × height) = lbh
= 12 \times 8 \times 6 = 576 \text{ cubic cm}

(iii) Diagonal of a cuboid = \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 8^2 + 6^2}
= \sqrt{144 + 64 + 36} = \sqrt{244} = 2\sqrt{61} \text{ cm.}

(iv) Length of cuboid = \frac{Volume}{Breadth \times Height} = \frac{v}{b \times h}
(v) Breadth of cuboid = \frac{Volume}{Length \times Height} = \frac{v}{l \times h}
(vi) Height of cuboid = \frac{Volume}{Length \times Breadth} = \frac{v}{l \times b}

(vii) Volume of cube = (side)³
= 12³
= 1728 \text{ cubic cm}

Cube : All sides are equal = 12 \text{ cm}

(viii) Sides of a cube = \sqrt[3]{Volume}
= \sqrt[3]{1728} = 12 \text{ cm}
Diagonal of cube = $\sqrt[3]{3} \times \text{(side)} = \sqrt[3]{3} \times 12 = 12\sqrt[3]{3} \text{ cm}$

Total surface area of a cube = $6 \times \text{(side)}^2 = 6 \times 12^2 = 864 \text{ sq.cm}$

**Right Circular Cylinder**:

(i) Area of curved surface

= (perimeter of base) $\times$ height

= $2\pi rh$ sq. unit

= $2 \times \frac{22}{7} \times 7 \times 15 = 660 \text{ sq. cm}$

(ii) Total surface area = area of circular ends + curved surface area

= $2\pi r^2 + 2\pi rh = 2\pi r(r + h)$ sq. unit

= $2 \times \frac{22}{7} \times 7(15 + 7)$

= $2 \times 22 \times 22$

= $968 \text{ sq. cm.}$

(iii) Volume = (area of base) $\times$ height

= $(\pi r^2) \times h = \pi r^2h$

= $\frac{22}{7} \times 7 \times 7 \times 15 = 2310 \text{ cubic cm.}$

(iv) Volume of a hollow cylinder = $\pi R^2h - \pi r^2h$

= $\pi h(R^2 - r^2) = \pi h (R + r)(R - r)$

= $\pi \times \text{height} \times \text{(sum of radii)} \times \text{(difference of radii)}$

Here $R$, $r$ are outer and inner radii respectively and $h$ is the height.

**Cone**:

(i) In right angled $\triangle OAC$, we have

$l^2 = h^2 + r^2$

(here $r = 35 \text{ cm}$, $l = 37 \text{ cm}$, $h = 12 \text{ cm}$)
Or, \( l = \sqrt{h^2 + r^2} \)

\[ h = \sqrt{l^2 - r^2} \quad r = \sqrt{l^2 - h^2} \]

where \( l \) = slant height, \( h \) = height, \( r \) = radius of base

(ii) Curved surface area = \( \frac{1}{2} \times \text{(perimeter of base)} \times \text{slant height} \)

\[ = \frac{1}{2} \times 2\pi r \times l = \pi rl \text{ sq. unit} \]

\[ = \frac{22}{7} \times 35 \times 37 = 4070 \text{ sq. cm} \]

(iii) Total surface area \( S \) = area of circular base + curved surface area

\[ = (\pi r^2 + \pi rl) = \pi r(r + l) \text{ sq. unit} \]

\[ = \frac{22}{7} \times 35(37 + 35) = 7920 \text{ sq. cm} \]

(iv) Volume of cone = \( \frac{1}{3} \) (area of base) \times height

\[ = \frac{1}{3} (\pi r^2) \times h = \frac{1}{3} \pi r^2 h \text{ cubic unit} \]

\[ = \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12 \]

\[ = 15400 \text{ cubic cm} \]

**Frustum of Cone:**

(v) Volume of frustum = \( \frac{1}{3} \pi h(R^2 + r^2 + Rr) \) cubic unit

(vi) Lateral surface = \( \pi l(R + r) \)

where \( l^2 = h^2 + (R - r)^2 \)

(vii) Total surface area = \( \pi [R^2 + r^2 + l(R + r)] \)

\( R, r \) be the radius of base and top the frustum

ABB’A’ h and l be the vertical height and slant height respectively.

**Sphere:**
(i) Surface area = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2 = 1386 \text{ sq. cm}$$

here, $d = 21 \text{ cm}$ \Rightarrow r = 10.5 \text{ cm}

(ii) Radius of sphere = $\sqrt{\frac{\text{Surface area}}{4\pi}} = \sqrt{\frac{1386 \times 7}{4 \times 22}} = 10.5 \text{ cm}$

(iii) Diameter of sphere = $\sqrt{\frac{\text{Surface area}}{2\pi}} = \sqrt{\frac{1386 \times 7}{22}} = 21 \text{ cm}$

(iv) Volume of sphere $V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = \frac{1}{6} \pi d^3$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \times 21 = 4831 \text{ cubic cm}$$

(v) Radius of sphere = $\sqrt{\frac{3}{4\pi} \times \text{Volume of sphere}}$

(vi) Diameter = $\sqrt{\frac{6 \times V}{\pi}}$

(vii) Volume of spherical ring = $\frac{4}{3} \pi (R^3 - r^3)$

(viii) Curved surface of hemisphere = $2\pi r^2$

(ix) Volume of hemisphere = $\frac{2}{3} \pi r^3$

(x) Total surface area of hemisphere = $3\pi r^2$

**Note**: $V = \text{volume}$, $A = \text{area}$, $h = \text{height}$, $b = \text{base}$, $b = \text{breadth}$, $d= \text{diameter}$, $R = \text{outer radius}$, $r = \text{inner radius}$, $\pi = \frac{22}{7} = 3.142$, $a = \text{side}$.

**Prism and Pyramid**

**Prism**

1. **Solid**: Bodies which have three dimensions in space are called solid. For example, a block of wood.

   A body, which has the three dimensions length, breadth and height, is a solid, whereas a rectangle with its two dimensions (length and breadth) is not a solid.

2. **Prism**: A prism is a solid, bounded by plane faces of which two opposite sides known as bases are parallel and congruent polygons.

3. **Base**: The congruent and parallel faces of a
prism are called its bases.

The other faces of a prism can be either oblique to the faces or perpendicular to them.

4. **Right prism**: A right prism is a prism in which lateral sides are rectangular or perpendicular to their bases.

5. **Lateral faces**: The side faces of a prism are called its lateral faces.

6. **Lateral surface area**: The area of all the lateral faces of a prism is called its lateral surface area.

**Note**: In a right prism having polygons of n sides as bases.

(i) the number of vertices = 2
(ii) the number of edges = 3n
(iii) the number of lateral faces = (n + 1), and
(iv) all the lateral faces are rectangular.

**Formulae**

| (i)  | Volume of a right prism = (Area of its base) x height |
| (ii) | Lateral surface area of a right prism = (perimeter of its base) x height |
| (iii) | Total surface area of a right prism = (lateral surface area) + 2(area of the base) |

**Pyramid**

1. **Pyramid**: A solid of triangular lateral sides having a common vertex and plane rectilinear bases with equal sides is called pyramid.

2. **Height of the pyramid**: The length
of perpendicular drawn from the vertex
of a pyramid to its base is called the
height of the pyramid.

The side faces of pyramid form its lateral surface.

3. **Regular pyramid** : If the base of a pyramid is a regular figure i.e., a polygon
   with all sides equal and all angles equal, then it is called a regular pyramid.

4. **Right pyramid** : If the foot of the perpendicular from the vertex of a pyramid
to its base is the centre of the base then it is called a right pyramid.

5. **Slant height of a regular right pyramid** : The slant height of a regular right
   pyramid is the length of the line segment joining the vertex to the mid-point of
   one of the sides of the base.

6. **Tetrahedron** : When the base of a right pyramid is a triangle, then it is called
   a tetrahedron.

7. **Regular tetrahedron** : A right pyramid with equilateral triangle as its base is
   called a regular tetrahedron.